Assignment for class 12 (Mathematics)

General direction for the candidate: Notes provided must be copied in maths copy and then homework should be done in the same copy.

Area Of a Triangle & determinant

- If (x_1, y_1) ,(x_2,y_2) and (x_3,y_3) be the vertices of triangle ,then

  Area of triangle= the absolute value of $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

  Note:

- **Area is a positive quantity, we always take the absolute value**
- **If area is given, use both positive and negative value of the determinant for calculation . (<u>Refer to the</u> <u>video on you tube link given to you.Ex 4.3. *Question no 6 solved*)</u>**
- **Area of the triangle formed by three collinear points is zero.(<u>refer to the video on you tube link given to you .Ex 4.3.Question no 4 solved)</u>**

  Condition of collinearity **of three points** (x_1, y_1) ,(x_2,y_2) and (x_3,y_3)

  $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}=0$

Exercise 4.3 Q1.ii) Area of triangle whose vertices are (2,7), (1,1) and (10,8)

= absolute value of $\frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

= absolute value of $\frac{1}{2}\left[(2\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7\begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix})\right]$

= absolute value of $\frac{1}{2}$ [2(-7) -7(1-10) +1(8-10)]

= absolute value of $\frac{1}{2}$ [(-14) -7(-9) +1(-2)]

= absolute value of $\frac{1}{2}$ (-14 +63 -2)

= absolute value of $\frac{1}{2}$ (-14 +63 -2)

= absolute value of $\frac{1}{2}$ (47)

=23.5 sq. units

Q9. If the points (a,0) , (0,b) and (1,1) are collinear , prove that a +b= ab/

$$\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow a\begin{vmatrix} b & 1 \\ 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & b \\ 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ a(b-1) +1(-b) =0

$\Rightarrow$ ab -a -b=0

$\Rightarrow$ a +b=ab proved

Homework: Exercise 4.3. QNo2, 5, 7,8

Adjoint & Inverse of a square matrix

- ***Adjoint of a square matrix is transpose of matrix of the co-factors of the given matrix***

Method to find adjoint of square matrix of order 3:

Let A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ **M**atrix formed by co-factors= $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

adj **A =Transpose of** $\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$ **=** $\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ **(Note:** ***adj is adjoint of A)***

EX 4.4 13i) Example :Find adjoint of matrix ***A=*** $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

$A_{11}=3$, $A_{12}=-12$, $A_{13}=6$, $A_{21}=1$, $A_{22}=5$, $A_{23}=2$, $A_{31}=-11$, $A_{32}=-1$, $A_{33}=5$

adj A= Transpose of $\begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}$ **=** $\begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$ **(verification part try yourself)**

Method to find adjoint of a square matrix of order 2 ;

Let A=$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ Matrix formed by cofactrs = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

adj A = Transpose of $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

Note: ***For square matrix of order 2 the adj A can also be obtained by interchanging*** a_{11} ***&*** a_{22} ***and changing signs of*** a_{12} ***&*** a_{21}

Question 3.i) Write the adjoint of matrix $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

adjoint of $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

Homework : Exercise 4.4 Q3.ii), Q7 iii), Q.15

Singular and Non Singular matrix:

- **A square matrix is said to be singular if |A|=0**
- **A square matrix is said to be non singular if |A|≠0**

Q1.i) If a matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular , find x.

= determinant of $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix} = \begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix} = 0$ (singular matrix)

$\Rightarrow$ (5-x)(4) -(2)(x+1)=0

$\Rightarrow$ 20-4x-2x-2=0

$\Rightarrow$ 6x=18

$\therefore x = 3$

Homework: Exercise 4.4 Q1iii), Q2.ii) Q10 ii)

Note : ***Theorems are proved in the video link provided to you by school***

Theorem 1

If A is a square matrix of order 3, then A(adj A) = |A|I_3=(adj A) A

Note: *A(adj A) = |A|I_n=(adjA) A* ***is true for any square matrix A of order n***

Q13i) $A=\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$ **$A_{11}=3$,$A_{12}=-12$, $A_{13}=6$,$A_{21}=1$ $A_{22}=5$, $A_{23}=2$, $A_{31}=-11$, $A_{32}=-1$, $A_{33}=5$**

adj A = Transpose of $\begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}$ **=** $\begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$

= A (adj A) = $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$ **=** $\begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$ **=27** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **=27 I_3 =**

= (adj A)A = $\begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$ **=** $\begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$ **=27** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **=27 I_3**

= $\therefore$ **A (adj A)=(adj A)A =|A| I_3 (Verified)**

Homework Exercise 4. Q11.ii),Q13ii)Q14

Theorem 2. Every invertible matrix has a unique inverse. (proved in the video link provided by school)

Theorem 3. A square matrix is invertible if it is non-singular. (proved in the video link provided by school)

- *Above theorem provides a theorem for finding inverse of square matrix.*
  Given : A is a square Matrix , & If |A|≠0 then

  $$A^{-1} = \frac{1}{|A|} \text{adj } A$$

- If A is a square matrix of order n & there exist square matrix B of order n such that AB $=I_n$ or BA= I_n, then B= A^{-1}

Q4.iv) Write A^{-1} for A= $\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$

|A|= (6)-(-1) = 7

since |A| ≠0 , A is non-singular & its inverse exist

adj A= $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

therefore $A^{-1} = \frac{1}{|A|}$ adjA = $\frac{1}{7}\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$

Q.18 A= $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & 0 \\ 0 & -1 & 2 \end{bmatrix}$ $|A| = 1\begin{vmatrix} 1 & 0 \\ -1 & 2 \end{vmatrix} - 0 \begin{vmatrix} -2 & 0 \\ 0 & 2 \end{vmatrix} + 2\begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix}$ = 2-0 +4=6

$|A| \neq 0 \Rightarrow A^{-1}$ exist $\Rightarrow$ A_{11} =2 , A_{12}=4, A_{13}= 2, A_{21}=- 2, A_{22}=2, A_{23}=1, A_{31}=-2, A_{32}=-4, A_{33}=1

adj A= $\begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 1 \\ -2 & -4 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 & -2 \\ 4 & 2 & -4 \\ 2 & 1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -2 & -2 \\ 4 & 2 & -4 \\ 2 & 1 & 1 \end{bmatrix}$ Ans

Homework:Exercise4.4 Q.16i) Q.17, Q19.ii)